

STATISTICS COLLOQUIUM

WEDNESDAY, FEBRUARY 20, 2008

TALK: 11:00 AM — SCIENCE CENTER RM. 705

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“Bootstrap in some Non-standard Problems”

ABSTRACT

The talk will consider some issues with the consistency of different bootstrap methods for constructing confidence intervals in two non-standard problems characterized by shape restricted estimation. The study of consistency of bootstrap methods in these problems is motivated by the problem of estimating dark matter distribution in Astronomy.

The Grenander estimator, the nonparametric maximum likelihood estimator of an unknown non-increasing density function f on $[0, \infty)$, is a prototypical example of a class of shape constrained estimators that converge at rate cube-root n . We focus on this example and illustrate different approaches of constructing confidence intervals for $f(t_0)$, for $0 < t_0 < \infty$. It is claimed that the bootstrap estimate of the sampling distribution of the Grenander estimator, when generating bootstrap samples from the empirical distribution function (e.d.f.) or its least concave majorant (the maximum likelihood estimate), does not have any weak limit, conditional on the data, in probability.

The other problem arises in Astronomy and is similar to the Wicksell's Corpuscle problem (1925, *Biometrika*). We observe (X_1, X_2) , the first two co-ordinates of a three dimensional spherically symmetric random vector (X_1, X_2, X_3) . Interest focuses on estimating F , the distribution function of $X_1^2 + X_2^2 + X_3^2$. This gives rise to an inverse problem with missing data. We propose two estimators of F and derive their limit distributions. Although the normalized estimators of F converge to a normal distribution, the non-standard asymptotics involved with the non-standard rate of convergence $(n/\log n)^{1/2}$, cast doubt on the consistency of bootstrap methods. We focus on bootstrapping from the e.d.f. of data, and show that the estimates can be bootstrapped consistently. A comparison of the two examples sheds light on some of the reasons for the (in)-consistency of bootstrap methods

This is joint work with Moulinath Banerjee and Michael Woodroffe.