STATISTICS COLLOQUIUM

MONDAY, FEBRUARY 07, 2011
TALK: 4:00 PM - SCIENCE CENTER RM. 309
RECEPTION: 5:15 PM - SCIENCE CENTER, 7TH FLOOR

Evaluating Multi-level Shrinkage Procedures

Carl N. Morris
Statistics Department
Harvard University

ABSTRACT

Statisticians regularly use normally distributed sample means to estimate their corresponding population means $\mu = (\mu_1,\ldots,\mu_k)$, a procedure that is unbiased, minimax, and has constant risk for the sum of mean squared errors (MSE). Despite these desirable properties, the sample mean vector is inadmissible for $k \geq 3$, as Charles Stein proved (1955). For example, for all μ, the James-Stein shrinkage estimator has a smaller summed MSE. No procedure can dominate for every μ for every component, or offer acceptable shrinkage patterns when the component means have substantially unequal variances, or provide successful component-wise interval estimates. The fault lies in requiring unattainable evaluations to hold for every μ.

The solution has been to extend the model above to "Model-II", which additionally views the μ_j as random effects at Level-II, governed by their own parameterized distributions. In practice statisticians often use Model-II (and its generalizations) with vague priors on the hyper-parameters to obtain Bayesian estimates and intervals. Decision theorists have given little attention to Model-II evaluations of such procedures, i.e. verifying, for the range of Level-II unknown hyper-parameters, which priors and which resulting Bayes estimators and intervals provide globally smaller component-wise MSEs and adequate coverage's.

This talk reviews the just mentioned history, including how Stein's harmonic prior (SHP) stands out among non-informative priors in its Model-II evaluations. More recently, the adjustment for density maximization (ADM) procedure is seen to improve on the MLE for shrinkage factor point and interval estimation, with minimal added computational complexity. Illustrative data sets include those from hospital profiling. Much remains to be done on evaluations of priors and procedures for Model-II and its extensions. Some results are available from recent work and forthcoming papers with former and current Harvard statistics students S. Michalak, R. Tang, M. Lysy, and J. Kelly.