



Estimation of Overflow Probabilities for Models with Heavy Tails and Complex Dependencies

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Events such as loss packets in queuing systems and bankruptcy probabilities for insurance companies are rare but consequential. The likelihood of such events can often be linked to the maximum of a stochastic process. We refer to such quantities as overflow probabilities—due to their connection to queues. Current Monte Carlo algorithms for efficient evaluation of overflow probabilities often assume features such as light tails or independence among increments. Our contribution lies in the design and the rigorous statistical analysis of estimators that are applicable to a class of problems that exhibit heavy-tailed features or arbitrary dependence structures.

After describing our results in settings involving heavy tails and models with short-range dependence, we concentrate on the third part of the dissertation where discrete-time Gaussian processes with negative drift and arbitrary dependency structures are considered. In great generality overflow probabilities to a level b for such processes converge to zero exponentially fast in b . Under mild assumptions on the marginal distributions of the process and no assumption on the correlation structure, we develop an importance sampling algorithm, called Target Bridge Sampler (TBS) that estimates the required overflow probability with polynomial complexity in b and is very easy to implement. This is the first provably efficient estimator for overflow probabilities of Gaussian processes that might exhibit long range dependence. TBS can also be used to generate exact samples for such processes given a large overflow. Numerical example is discussed for the case of fractional Brownian noise driven processes. It validates our theoretical findings and delivers vast superior performance to existing methods.